



## **Self-organization of the tribosystem under non-stationary conditions of friction from the standpoint of deformation-wave representations**

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### **Abstract**

Mechanisms of structural adaptation of contact surfaces and lubricating materials during friction with the dominance of deformation processes in tribocontact have been analyzed. The purpose of the work was to model the elastic-plastic properties of dissipative structures taking into account the anisotropic properties of the surface layers of the friction pairs and the boundary layers of the lubricating material. The modeling took into account the structural state of the latter formed due to heating and saturation with wear products, along with the physical and mechanical interaction of this layer with the outer surface of the part. An algorithm for determining the distributed tangent force along the length of the boundary layer of the lubricating material adjacent to the part has been developed based on the hypothesis of the wave-like state of the surface layer of the lubricating material on an absolutely flat (non-deformed) rough surface. Herein, under the action of tangent forces, the strip of lubricant is subject to horizontal compression and transverse movement. The distributed tangent stress along the length of the adjacency of the layer of densified lubricant to the part causes micro-slipping of the layer. Amplitude horizontal displacements of the boundary of the lubricant layer are determined when the beam-film is loaded with longitudinal stresses, which leads to partial disorientation of the film and loss of its originally rectilinear structured form, the transition of the lubricant layer to the state of the wave surface of a sinusoid shape. Also, a procedure for calculation of tangent forces causing the loss of elastic stability of the lubricant boundary layers resulting in the direct mechanical destruction of the lubricant boundary layer in the slipping zone of the contact surface is proposed based on the elastic-frictional interaction of this layer with the near-surface layer of the metal.

**Key words:** wear, self-organization, lubrication, deformation, boundary layers, tangent forces, rough surface.

### **Introduction**

In terms of energy, contact interaction during friction can be represented as a set of processes of surface interaction with the flow of mechanical energy, whose law of distribution on the friction plane is adequate to the diagrams of tangent stresses and relative sliding speeds. The action of such source provokes changes in the internal structural and energy states of the contact layers of materials and the microgeometry of the contact, while most of the flow of mechanical energy is transformed into heat. The non-dissipative component of the mechanical energy is spent on the formation of secondary structures and wear [1].

The resistance of the lubricating film to mechanical destruction due to an increase in the shear rate gradient is a determining factor that ensures the normal performance of friction pairs under critical conditions. The destruction of the lubricating film during friction is one of the leading factors causing the intensification of energy processes occurring in the contact zone. First of all, this manifests itself in the violation of the structural adaptation of the contact surfaces and lubricant under critical friction conditions and the destruction of previously formed metastable structures [2].

Under the conditions of loading of flat surfaces by tangent forces with significant overloads (over the level of critical forces), flat surfaces are loaded by sliding forces and lose their original shape taking the shape of wavy surfaces, i.e. they acquire, in addition to longitudinal, noticeable transverse deformation.



Many effects under friction and wear of interacting surfaces in the presence of lubricant in the contact zone can be explained in terms of a deformation-wave approach, [according to which](#), wave-like deformations appear in the zone located in front of the moving part. Tangent forces applied through a moving part (stamp) cause deformation-wave processes, in both stationary and moving parts. The deformation waves appear on the front surfaces of the interacting half-spaces of the contact surfaces and attenuate in the surface layer of the material at a length of 2-3 deformation half-waves. During the movement of the stamp, the deformation wave precedes the moving part by 2-3 half-waves.

In such a case, questions about the deflection arrow of the wave-like surface, the length of the deformation section of the beam-strip of the densified lubricating material, and the deformation half-wave length remain open. To determine these parameters, additional information is needed, which can be obtained on the basis of data from the energy indicators of interacting parts in the contact zone and data on the attenuation of the tribosystem's total energy along the deformation wave length.

### Literature review

The service life, reliability in operation, structural strength, technical and economic indicators of the operation of parts of machines and mechanisms are largely determined by the mechanical properties of the steels and alloys of which they are made. Improving the quality of the surface of parts of machines and mechanisms increases the service life, especially when determined by the mechanical and tribotechnical properties of the surfaces [3].

The main changes in the material during friction are localized in a thin (up to several micrometers) surface layer. Localization of stresses and their impulsive character during friction lead to the generation of deformation defects such as point defects, dislocations, slip bands, etc. [4]. In [5], the peculiarities of the microdeformation of the surface layers and the mechanisms of titanium wear have been investigated. As shown, a decrease in the workaction of elastic-plastic deformation leads to a decrease in the resistance to the destruction of titanium during friction by 2-3 times, compared to the initial state. Wear occurs due to the formation of cracks and brittle destruction of surface layers. After the abrasion of the flooded layer, the plastic deformation of microprotrusions predominates.

Paper [6] considers thermodynamic systems where only deformation processes or tribochemical reactions dominate. Herein, irreversible damage to the metal is associated with the accumulation and interaction of structural defects, chemical or electrochemical interaction between the metal and the environment, and the formation of new surfaces during dislocation discharge. The thermodynamic system is homogeneous and isotropic, and fatigue processes occur under isobaric-isothermal conditions.

Initially, a sign-changing contact load plastically deforms the near-surface layers, causing their strengthening to a state that is akin to active static strengthening. The following reversible loads are associated with elastic re-deformation of the surface layers. The transition to the regime of elastic cyclic deformation is caused, on the one hand, by the strengthening of the metal and, on the other hand, by a decrease in the effective amplitude of the sign-alternating load due to the easing friction conditions when wear products appear. Further, the processes develop by the usual fatigue laws [7]: previously weakened layers are first disordered to a certain level, and after that (under pre-deformation conditions), their re-strengthening takes place, and these processes are periodically repeated [8]. Fatigue processes are accompanied by intensive formation of vacancies, the coalescence of which leads to the appearance of pores and microcracks [9].

The surface levels of the half-spaces in the tribocontact zone cannot move freely along the joint surface of the parts and meet on their way horizontal connections in the form of strong tangent forces which prevent a free shift of the elements of deformation layers. That is why, when tangent forces reach certain critical values, such layers become capable of bending, which gives them the ability to implement longitudinal stresses [10]. Under such a mechanism of interaction of parts, their surface layers either lose the longitudinal stability, or fall under the influence of cyclic elastic deformations which disappear after the load is removed. Note that during the interaction of contacting parts, their surface layers (dissipative structures) are initially prone to plastic deformations due to the action of the dissipating component of the kinetic energy of the part movement and converting it into thermal energy. With further movement of the parts, surface strengthening of the outer layer occurs with a softer (pliable) base of the lower material layers of the contacting surfaces. It should be noted that the operation of friction pairs in the elastohydrodynamic and boundary lubrication mode leads to the densification of the lubricating layer on the contact surfaces activated by friction and the formation of boundary films of the lubricant in the zones adjacent to the surface layers of the parts as a result of structural adaptation. The loaded layers of the parts become covered with a densified and viscous boundary layer of the lubricant, whose consistency thickens due to the acquisition of non-Newtonian properties by the anisotropic layers, the appearance of small products of parts' wear and chemical interaction of metal surfaces and components of the lubricant. As a result of such interaction, the formation of a metal layer (beam-strip) and an elastoplastic coating of lubricant material takes place on the densified surface.

If we adhere to the idea of the formation of dissipative structures on metal parts under friction, then the parts under consideration represent an anisotropic medium that bears the load in the boundary layers (a densified beam-strip with an upper elastic-plastic layer of lubricant) from the tangent forces in the surface layer. These two outer layers lie on the inner layers of the parts ("elastic" base) through longitudinal and transverse connections.

Thus, the evaluation of the elastic-plastic properties of lubricating anisotropic layers under the conditions of tangent stresses is an important task in modeling the processes of physical-mechanical interaction of the base metal, oxides, and boundary layers of the lubricating material.

**Purpose**

To model the elastic-plastic properties of dissipative structures taking into account the anisotropic properties of the surface layers of friction pairs and the boundary layers of lubricants.

**Modeling of elastic wave-like deformation processes under loading of contact surfaces by tangent forces during friction**

Considering an anisotropic (three-layer) zone for interacting parts, we assume that the acting tangent force does not exceed certain critical force for the beam-strip  $T_1 < T_{cr1}$ . This means that the part that has a densified oil layer is located on an absolutely flat (non-deformed) rough surface. In this case, the beam-strip is under the action of limiters of transverse movements caused by the transverse connections of the elastic base. We also assume that there is a reactive tangent force between the densified layer of lubricant and the beam-strip

$$T = \Delta T + ql + cu(x), \tag{1}$$

where  $q = f_{ad}Pb$  is the limit force of friction per unit length of the strip on the densified lubricant ( $l$  – is a number corresponding to the length of the beam-strip);  $P$  – is the vertical load on the part;  $b$  – is the width of beam-strip;  $u = u(x)$  is the longitudinal replacement of the densified lubricant;  $f_{ad}$  – is the coefficient of contact friction (adhesion) between the densified layer of lubricant (with wear particles of parts) and the beam-strip (surface layer of the part);  $c$  – is the longitudinal stiffness of the densified layer.

Using the values of dimensionless parameters according to [12] and assuming that  $T_1 < T_{1cr}$  where  $T_{1cr}$  is the critical force for the beam-strip, we can obtain the dependence of the critical force on the length of the beam-strip. So, if

$$\bar{l} = \frac{l^4}{\pi} \sqrt{\frac{k}{E_c I}} \tag{2}$$

and

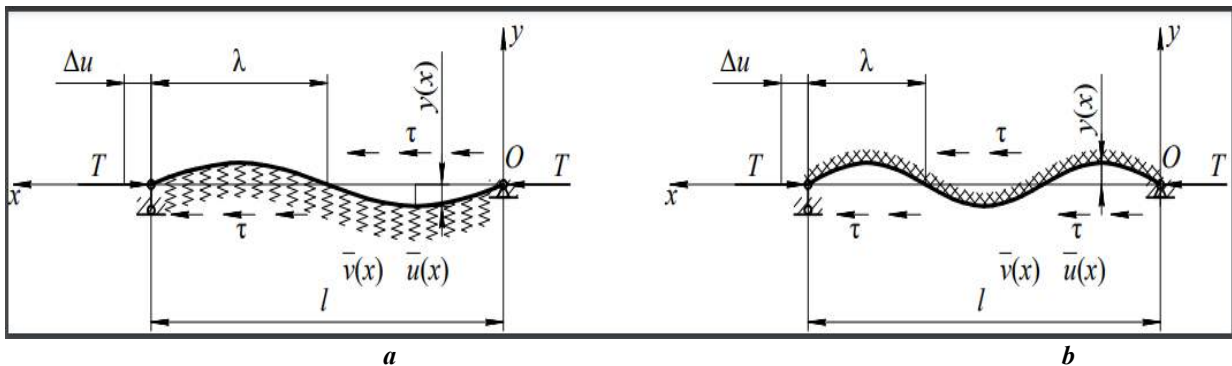
$$\bar{T} = \frac{T}{\sqrt{k E_c I}} \tag{3}$$

then by [12]

$$T_{cr} \approx 2\sqrt{k E_c I}. \tag{4}$$

In expressions (1) – (4),  $\Delta T$  – is the excessive tangent force (the degree of overload  $T$  compared to  $T_{cr}$ );  $k$  – is the coefficient of stiffness of the elastic base by Winkler;  $E_c$  – is the modulus of elasticity of the surface layer of the part.

Considering the anisotropic three-layer problem (densified lubricant, densified surface layer of metal, and elastic base), we assume that there is an elastic-frictional connection between the metal layer and the densified layer of lubricant (Fig. 1). For such a task, the critical Euler force is determined by the coefficient of friction for liquid (or boundary) lubrication and does not depend on the length of the beam-strip (at least when the number of half-waves  $n$  on the outer side of the beam-film is  $n \geq 3$  [1]). Therefore, significant inaccuracies in the formulation and solution of the problem are not allowed: the length of the beam-film  $l$  can be determined as the limit length under the conditions of relative mutual slippage for the beam-film with the three-layer anisotropic surface.



**Fig. 1. Longitudinal bending of a beam-film: a – shape with two half-waves; b – shape with three half-waves**

Let us consider a densified layer of lubricant on the surface of the part, assuming that this layer undergoes a mechanical-thermal destruction, densifies, becomes saturated with wear products, and interacts with the outer flat film strip of the part. This layer can be called the outer layer of the part that takes on itself the external normal and tangent loads on the friction pair.

The normal load is applied to the place where the parts join, whereas the tangent load acts in front of the moving part (stamp). Thus, the densified layer of lubricant, which is attached to the outer layer of metal, can be considered a thin elastic-plastic plate subjected to tangent forces in front of the stamp (Fig. 1).

Then the differential equation that describes the longitudinal bending of such a beam-strip (plate) under the action of tangent forces, has the form:

$$E_c I y^{IV} + T y'' + ky = 0, \quad (5)$$

where  $E_c$  – is the reduced modulus of elasticity of the densified layer of lubricant;  $T = \Delta T + ql + cu(x)$  is the tangent force applied to the plate from the surface layers of the lubricant, which is spent on the loss of longitudinal stability of these layers and their accumulation of irreversible deformations;  $\Delta T$  – is an excess tangent force, which is spent on the accumulation of final wave-like deformations in the surface layers;  $T_{cr}$  – is Euler's critical force for a beam-strip, at which the beam-strip loses its longitudinal stability;  $k$  – is the coefficient of stiffness of the elastic base of the part for the densified lubricant (herein, the reactive forces in the bases between the layers vary sinusoidally);  $I$  – is the moment of inertia of the cross-section of the hardened lubricant layer.

In expressions (5) and (15), the force  $T$  is such a longitudinal force that corresponds to the case when anisotropic layers of beam-films of parts and the elastic base do not meet the hypothesis of flat sections (that is, they undergo deplanation of flat sections).

The boundary conditions of the beam-film have the form:

$$\begin{aligned} y(0)=0; y''(0)=0; \\ y(l)=0; y''(l)=0, \end{aligned} \quad (6)$$

Considering the support of the rod as hinged, let us consider the set of functions:

$$y_n(x) = A_n \sin \frac{\pi n x}{l}, \quad \text{where } n = 1, 2, 3, \dots \quad (7)$$

Let us denote  $\alpha_n = \frac{\pi n x}{l}$  where  $n$  is the number of half-waves on a curved beam of length  $l$ .

The set of functions (7) represents a system of the eigenfunctions of problem (5). For a beam-strip lubricant on a part of length  $l$ , two cases of lubrication can be calculated: 1 – the lubricant strip lies on an elastic base ( $k \neq 0$ ); 2 – the lubricant strip hinges on a part without an elastic base ( $k=0$ ).

Turning to equation (7) and using the notion of deflection functions as the eigenfunctions of the problem

$$y(x) = A_n \sin \alpha_n x, \quad (8)$$

then substituting (8) into equation (5), we obtain:

$$T_c = E_c I \alpha_n^2 + \frac{E}{2\alpha_n}. \quad (9)$$

The minimum (9) is found from the equation

$$\frac{\partial T_c}{\partial \alpha_n} = 0. \quad (10)$$

which yields

$$\alpha_n = \sqrt[3]{\frac{E}{4E_c I}}. \quad (11)$$

As follows from [13], the stiffness coefficient of the elastic base

$$k = \frac{E \alpha_n}{2} \quad (12)$$

or

$$k = \frac{E}{2} \cdot \sqrt[3]{\frac{E}{4E_c I}} \quad (13)$$

Based on (12),  $\alpha_n$  can be interpreted as an unknown number of half-waves per film section of length  $\pi$ . In this case, we present  $\lambda_n = \pi \sqrt[3]{\frac{4E_c I}{E}}$  as the half-wave length of the section of hardened lubricant that lost its stability.

The length of the film section that lost stability is equal to

$$l = n \lambda_n. \quad (14)$$

The critical load for a beam-film (an analogue of the Euler force) is determined based on (4) and (13).

Therefore, the initial growth rate of  $A_n$  (increase in corrugations) becomes less intense as they are filled, their longitudinal wave-like deformation is preserved, the hypothesis of a wave-like surface layer is realized, and the Winkler-Fuss hypothesis of a beam on an elastic base remains valid.

Turning to the function  $y(x)$ , we will treat  $A_n$  as the arrow of the beam-film deflection under the action of the tangent force (friction force). Initially, the deformed beam-film has a corrugated surface. In the process of further exfoliation of metal particles, their chemical interaction with oxide particles, and partial mechanical destruction of the boundary layers of the lubricant, the adjacent depressions between the corrugations will be filled with solid, heated to significant temperatures, metal wear elements, coked lubricant, and dust. At the same time, this process is reduced to the filling of the open elements of the corrugations with the mentioned sludge and the growth of an insignificant layer of a spot-strip, which under the influence of tangent force of liquid or semi-liquid friction also acquires a wave-like shape from the outside of the open corrugations. The height of these corrugations will be  $\bar{A}_n < A_n$ , and each subsequent layer of the oil film will fill the open corrugations of the deformed surface. Therefore, the initial rate of growth of  $A_n$ , that is the growth of corrugations, becomes less intense as they are filled,

their longitudinal wave-like deformation is preserved, the hypothesis of a wave-like surface layer is realized, and the Winkler-Fuss hypothesis of a beam on an elastic base remains valid.

Let us complicate the problem by taking into account that under the action of horizontal forces, the lubricant strip is subject to horizontal compression  $u(x)$  and transverse displacement  $y(x)$ . During longitudinal displacements of the lubricant strip, a compressive force acts on it, which consists of the overload force  $\Delta T$  (as a result of applying a force to the strip from another moving part with a dynamic coefficient);  $ql$  is the adhesion force of the rigid lubricant to the surface layer of the metal;  $cu(x)$  is the elastic force under horizontal loading of the lubricant (in the middle of the strip thickness). Taking into account such loading of the beam-strip, the horizontal tangent force that acts on the lubricant layer is determined as follows:

$$T = \Delta T + ql + cu(x), \quad (15)$$

where  $q=f_{ad} Pb$  – is the boundary limit value of the friction force (adhesion) between the metal and the solid lubricant layer;  $P$  – is the vertical load on the lubricant layer and the part;  $b$  – is the width of the lubricant layer;  $u=u(x)$  – is the longitudinal displacement of the lubricant layer along the part during their mutual slipping;  $l$  – is the length of the lubricant layer during its slipping (in other words, the boundary length of the lubricant layer in front of the moving part);  $c$  – is the stiffness (horizontal) of the lubricant layer during its slipping;  $u=u(x)$  – is the longitudinal (elastic) slippage of the lubricant layer during its deformation.

Note that the longitudinal stiffness  $c = E_c F$  is equal to the product of the elasticity modulus of the elastic lubricant layer and its cross section  $F = bh$  (where  $h$  is the lubricant layer thickness).

Then  $\Delta T' = cu(x)$  is the distributed tangent force along the length of the lubricant layer adjoining the part.  $\Delta T'$  is the reactive force preventing the loss of stability of the lubricant layer due to the action of the horizontal connection (in the zone under the stamp). Based on these considerations (and neglecting half the thickness of the lubricant film to exclude film compression processes), we can write a modified equation for the longitudinal stability of the lubricant film (instead of (5)) in the following form:

$$E_c I y^{IV} + T y'' + ky - E_c F u = 0. \quad (16)$$

This equation includes the transverse displacement of the beam-film  $y(x)$  and the longitudinal displacement of the film beam  $u(x)$ . Both of them arise under the action of the longitudinal force  $T$  and the bending deformation of the film beam. When the longitudinal force acts in the film beam, its movable end longitudinally shifts by the value  $l - u$ . Herein, the other end of the beam remains stationary. Then a nonlinear relationship is established between the longitudinal displacement of the beam-film and the transverse deflection. We find the longitudinal displacement of the movable end as the difference between the initial length of the film beam and the projection of the curved axis of the beam [14]:

$$u = l - \int_0^l \cos \varphi ds = l - \int_0^l \sqrt{1 - \left(\frac{\partial y}{\partial s}\right)^2} ds, \quad (17)$$

where  $\varphi$  is the angle between the tangent to the arc of the curved beam-film and the longitudinal axis  $Ox$ .

Let us expand the integrand in a series using the Newton binomial formula:

$$\sqrt{1 - \left(\frac{\partial y}{\partial s}\right)^2} = 1 + \frac{1}{2} \left(\frac{\partial y}{\partial s}\right)^2 + \frac{1}{8} \left(\frac{\partial y}{\partial s}\right)^4 + \dots \quad (18)$$

If we substitute into expression (18) instead of  $y(x)\{y(s)\}$  its value (8), then having performed needed transformations, we obtain:

$$u = \frac{\pi^2 A_n^2}{4l} + \frac{3}{64} \frac{\pi^4 A_n^4}{l^3} + \dots \quad (19)$$

With longitudinal displacements of the beam-film, an increment of the longitudinal force  $\Delta T'$  occurs, which can be expressed through the deflection arrow  $A_n$ :

$$\Delta T' = E_c F u = E_c F u(A_n) \quad (20)$$

Substituting the expression for  $u(A_n)$  in the differential equation (16) taking into account  $\Delta T'$  and performing simple mathematical transformations, we arrive at an expression containing the deflection arrow of the film beam (for transverse deformations):

$$E_c I \frac{\pi^4}{l^4} A_n + T \frac{\pi^2}{l^2} A_n + k A_n - \left( \frac{\pi^2 A_n^2}{4l} + \frac{3}{64} \frac{\pi^4 A_n^4}{l^3} \right) E_c I = 0. \quad (21)$$

In evaluating the contribution of each term to the result when calculating  $A_n$ , we assume that the influence of the term containing  $A_n^4$  is less significant than the others', so we write the result for  $A_n$  as follows:

$$A_n = E_c I \frac{4\pi^2}{l^3} + \frac{T}{l} + k \frac{4l}{\pi^2}. \quad (22)$$

Thus, we have obtained an expression for the deflection arrow of the beam-film on the elastic base of the part.

Since the film beam has bending, its transverse deflection is expressed by formula (22) through the modulus of elasticity of the first kind  $E_c$ , the moment of inertia of the beam cross section  $I$ , the compressive force that can reach its critical value  $T_{cr}$ , and the stiffness coefficient of the elastic base  $k$ . When  $n = 1$ ,  $A_n$  takes the greatest value.

Since the transverse deflections of the beam-film (and in some cases -- of a flat membrane)  $y(x)$  are presented in the form:

$$y(x) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{l}, \quad (23)$$

where  $\sin \frac{n\pi x}{l}$  are the eigenfunctions of the film beam, then the curved surface (axis) of the film beam in front of the moving part (stamp) can consist of one, two, three or more half-waves ( $n=1,2,3,\dots$ ).

The amplitude horizontal displacements  $u(A_n)$  are determined in the first approximation from formula (19), retaining only the first term in the expansion.

When the beam-film is loaded with longitudinal forces, it loses its original rectilinear shape and acquires the shape of a wave surface. In this case, the vertices-sinusoids of the film break away from the still flat base. Herein, one end of the beam film remains movable, and when the surface layer of the part is shortened, micro-sli of the densified lubricant layer relative to the part occurs due to longitudinal deformation of the film-strip. In this case, the adhesion forces of the densified lubricant layer keep the beam-film from shifting relative to the part, as a whole. This indicates that the complete displacement of the lubricant layer of length  $l$  will occur when

$$T \geq ql, \quad (24)$$

that is, when

$$l \leq \frac{T}{q}. \quad (25)$$

In this problem, we determine the largest value of the amplitude of bending (transverse) waves

The eigenmodes can have the form of sinusoids, as shown in Fig. 1. The amplitude horizontal displacements  $u(A_n)$  are determined in the first approximation from formula (19), taking only the first term in the expansion.

Based on these considerations, the length of the film-strip can be determined by formula (25).

Note that the tangent forces acting in anisotropic lubricant layers manifest themselves differently. In particular, in a densified lubricant layer adjacent to the metal, due to the physical and mechanical interaction of the base metal and oxides that fall into the boundary lubricant layer, the friction (adhesion) coefficient will be very high and exceed the dry friction coefficient in the outer layers of metal parts.

The coefficient of liquid friction between interacting parts is characteristic for the medium of oil films that have not been subjected to thickening and the ingress of solid particles, therefore it can be determined according to the theory of G.E. Svirsky [14] or other similar theories. Calculations by the Svirsky formulas are carried out taking into account the change in the kinetic energy of the material body under the assumption that the dissipation of the kinetic energy of a moving body occurs in the contact layer of small thickness  $h$ . The author [14] proceeded from the fact that the space between the rubbing bodies is filled with a mixture of gases, liquid vapors, and wear particles which are set in motion by the moving surfaces and provide additional viscous resistance expressed by the second term in (26). Then, for liquid lubrication, we obtain the following expression for the friction coefficient:

$$f = f_0(1 + \alpha_0^2 v^2) e^{-\alpha^2 v^2} + \frac{\xi}{hq_{sp}} v, \quad (26)$$

where  $f_0$  – is the coefficient of static friction;  $\alpha_0$  – is a function of the densities of bodies 1 and 2;  $\alpha$  – is a statistical coefficient associated with the probability law of the displacement of moving bodies;  $v$  – is the relative speed of moving bodies;  $\xi$  – is the viscosity coefficient of the liquid medium between the rubbing bodies; and  $q_{sp}$  – is the specific load.

Further, we will assume that the contact layer of thickness  $h$  consists of densified wear particles and chemically reacted lubricant particles.

### Modeling of elastic-frictional connections between the surface film-strip and the base material of the part

Let us consider a qualitatively different approach to supporting the outer lubricant layers on an elastic base in terms of the action of elastic-frictional connections on the outer flat and rough surface of the interacting half-spaces. Therefore, we will take into consideration such connections that also protect the beam-strip (film) from losing elastic stability and direct layer displacement.

First, consider an analogy between the metal outer layer of the part and part's main mass, and the densified layer (film) of lubricant and the main mass of the part. It can be argued that in both cases, there is an elastic-frictional connection between the surface film and the base.

Therefore, we will proceed from the elastic-frictional interaction of the film-strip of lubricant with the boundary layer of metal, which lies on the metal massif supported by an elastic base (from vertical connections). On the other hand, the layer of oil film is connected to the base metal by horizontal connections. This means that

we introduce in the consideration the elastic-frictional interaction of the film-strip and the non-deformed layer from the linear displacement and deformation of the densified layer of lubricant (along the horizontal axis  $Ox$ ). That is, the displacements of the left end of the beam occur under the action of the tangent force  $T$  (Fig. 2), which can be applied smoothly with the coefficient of application intensity in the range  $0 \leq \alpha \leq 1$ .

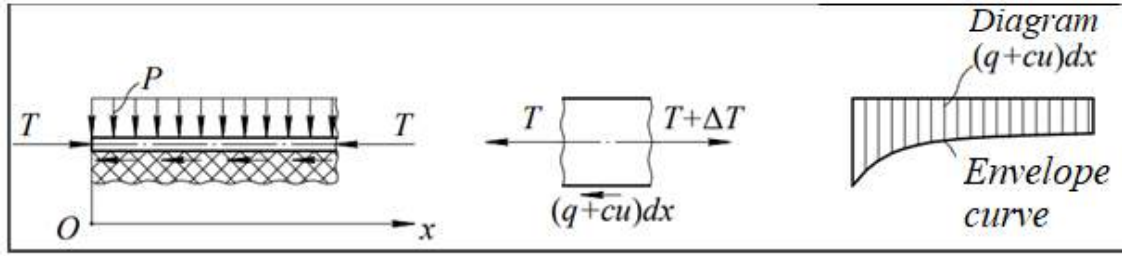


Fig. 2. Diagram of a beam-strip with horizontal connections

The dissipative structure of the lubricant densified layer (the beam-strip on an elastic base) is in equilibrium under the action of a system of horizontal forces. The equilibrium equation of an element is:

$$\frac{dT}{dx} - cu - q = 0, \quad (27)$$

where  $cu(x) = s$  (per length unit) – is the intensity of tangent connections;  $c$  is the stiffness coefficient of elastic connections;  $q$  – is the limit friction force per strip length unit.

Longitudinal force  $T$  can be expressed through deformation of the strip:

$$T = EF \frac{du}{dx}. \quad (28)$$

Then (22) may be written as:

$$\frac{d^2u}{dx^2} - \beta^2 u = \frac{q}{EF}, \quad (29)$$

where  $\beta = \sqrt{\frac{c}{EF}}$  is a parameter characterizing the relative stiffness of elastic connections.

Equation (29) can have a solution in the form

$$u(x) = C_1 sh\beta x + D_1 ch\beta x - \frac{q}{c}. \quad (30)$$

Arbitrary constants  $C_1$ ,  $D_1$ , as well as unknown length of the slipping zone  $a_1$ , are determined from three initial conditions

$$u(a_1, \alpha) = 0; u'(a, \alpha) = 0; u'(0, \alpha) = -\frac{\alpha T}{EF}. \quad (31)$$

The first two conditions relate to the right boundary of the zone, whereas the third one – to the initial strip cross section.

Then equation (29) acquires the form:

$$u(x, \alpha) = \frac{q}{c} \left[ \sqrt{1 + \left( \alpha \beta \frac{T}{q} \right)^2} ch\beta x - \alpha \beta \frac{T}{q} sh\beta x - 1 \right]. \quad (32)$$

In particular, at  $x = 0$ , that is at the beginning of the strip,

$$u(0, \alpha) = \frac{q}{c} \left[ \sqrt{1 + \left( \alpha \beta \frac{T}{q} \right)^2} - 1 \right]. \quad (33)$$

In equation (32), the force  $T$  can be considered applied suddenly ( $\alpha=1$ ), then (32) determines the initial displacement of the strip at  $x=0$ .

The equation is valid when the tangent force  $T$  is significantly less than the critical force for the strip,  $T \ll T_{cr}$ . That is, in this case, the hypothesis of flat sections is satisfied for the strip and longitudinal bending of the strip is not realized due to the action of tangent elastic connections.

## Conclusions

1. An algorithm for determining the distributed tangent force along the length of the boundary layer of the lubricating material adjacent to the part was developed taking into account the anisotropic properties of dissipative structures and the hypothesis about the wave-like state of the surface layer of the densified lubricating material on an absolutely flat (non-deformed) rough surface on an elastic base.

2. A procedure for calculation of the tangent forces leading to the loss of elastic stability of the boundary layers of the lubricating material, which causes the direct mechanical destruction of the boundary layer in the zone of slipping of the contact surfaces, is proposed taking into account the elastic-frictional interaction of the boundary layer of the lubricating material and the surface layer of the metal.

## References

1. Yakubov F.Ya. Synergetics and self-organization processes during friction and wear. *Printed scientific works: Modern technologies of engineering*, Kharkiv: NTU «KhPI», 2010, 5, P.122-133. [http://library.kpi.kharkov.ua/files/JUR/sutech5\\_2010.pdf](http://library.kpi.kharkov.ua/files/JUR/sutech5_2010.pdf)
2. Mnatsakanov R. G., Mikosianchyk O. A., Yakobchuk O. Ye., Tokaruk V. V. Forecasting of the maximum linear wear of contact surfaces in extreme friction conditions. *Problems of friction and wear*, 2018, 4 (81), C. 4 - 12. [https://doi.org/10.18372/0370-2197.4\(81\).13321](https://doi.org/10.18372/0370-2197.4(81).13321)
3. Meng D., Lv Z., Yang S. et al. A time-varying mechanical structure reliability analysis method based on performance degradation. *Structures*. 2021. Volume 34. P. 3247-3256. <https://doi.org/10.1016/j.istruc.2021.09.085>
4. He X., Gu F., Ball A. A review of numerical analysis of friction stir welding. *Progress in Materials Science*. 2014. Vol. 65. P. 1-66. <https://doi.org/10.1016/j.pmatsci.2014.03.003>
5. Pokhmursky V.V., Vynar V.A., Vasylyv Kh. B. et al. Peculiarities microstrain of surface layers and mechanisms wear  $\alpha$ -titanium under the influence of hydrogen. *Problems of Tribology*, 2013, № 2, P. 21-26.
6. Kadin Y., Sherif M. Y. Energy dissipation at rubbing crack faces in rolling contact fatigue as the mechanism of white etching area formation. *International Journal of Fatigue*, 2017, Vol. 96, P. 114-126. <https://doi.org/10.1016/j.ijfatigue.2016.11.006>
7. Mughrabi H. Cyclic Slip Irreversibilities and the Evolution of Fatigue Damage. *Metallurgical and Materials Transactions A*, 2009, Vol. 40. P. 1257–1279. <https://doi.org/10.1007/s11661-009-9839-8>
8. Mikosyanchyk, O.O., Mnatsakanov, R.H., Lopata, L.A. et al. Wear Resistance of 30KhGSA Steel Under the Conditions of Rolling with Sliding. *Materials Science*. 2019. Vol. 55, P. 402–408. <https://doi.org/10.1007/s11003-019-00317-9>
9. Wang X-S. Fatigue Cracking Behaviors and Influence Factors of Cast Magnesium Alloys. *Special Issues on Magnesium Alloys*. InTech. 2011. Available at: <http://dx.doi.org/10.5772/19075>.
10. Mi Ch. Surface mechanics induced stress disturbances in an elastic half-space subjected to tangential surface loads. *European Journal of Mechanics - A/Solids*, 2017, Vol. 65, P. 59-69 <https://doi.org/10.1016/j.euromechsol.2017.03.006>
11. Chawla N., Chawla K. K. *Metal Matrix Composites*. Springer Science+Business Media New York, 2013. 370 p. <https://doi.org/10.1007/978-1-4614-9548-2>
12. Luongo A., Ferretti M., Simona Di N. *Stability and Bifurcation of Structures: Statical and Dynamical Systems*. Springer Cham, 2023, 706p. <https://doi.org/10.1007/978-3-031-27572-2>
13. Wang X. Z., Yi J. T., Sun M. J. et al. Determination of elastic stiffness coefficients for spudcan foundations in a spatially varying clayey seabed. *Applied Ocean Research*, 2022, Vol. 128, P. 103336. <https://doi.org/10.1016/j.apor.2022.103336> [Get rights and content](#)
14. Uchitel A. D., Malinovsky Yu. A., Danilina G. V et al. Influence of parametric resonance on the mechanism of destruction of contacting surfaces during training and wearing. *Metal Journal*, 2018, №4. C. 65-73. <https://www.metaljournal.com.ua/assets/Journal/Uchitel.pdf>



**Маліновський Ю. О., Ільїна О. А., Власенков Д. П., Олійник С. Ю, Мікосянчик О. О.**  
Самоорганізація трибосистеми в нестационарних умовах тертя з позицій деформаційно-хвильових уявлень.

Проаналізовано механізми структурної пристосованості контактних поверхонь та мастильного матеріалу при терті при домінуванні деформаційних процесів в трибоконтаті. Мета роботи полягала в моделюванні пружньо-пластичних властивостей дисипативних структур з урахуванням анізотропних властивостей поверхневих шарів пар тертя та граничних шарів змащувального матеріалу. При моделюванні враховувався структурний стан граничного мастильного шару, набутий в результаті його нагрівання та насичення продуктами зносу, а також враховувалася фізико-механічна взаємодія цього шару із зовнішньою поверхнею деталі. Розроблено алгоритм визначення розподіленого дотичного зусилля по довжині примикання граничного шару мастильного матеріалу до деталі з урахуванням гіпотези про хвилеподібний стан поверхневого шару мастильного ущільненого матеріалу на абсолютно плоскій (недеформованій) шорсткій поверхні з пружною основою. Враховано, що під дією дотичних сил смужка мастильного матеріалу піддається дії горизонтального стиснення та поперечного переміщення. Розподілене дотичне зусилля по довжині примикання шару мастильного матеріалу до деталі обумовлює мікроковзання шару ущільненого мастильного матеріалу. Амплітудні горизонтальні зміщення граничного шару мастильного матеріалу визначаються при навантаженні балки-плівки поздовжніми зусиллями, що призводять до часткової дезорієнтації плівки і втрати своєї первісно прямолінійної структурованої форми, що сприяє переходу шару мастильного матеріалу в стан хвильової поверхні у формі синусоїда. Також запропоновано розрахунок визначення дотичних зусиль, спрямованих на втрату пружної стійкості граничних шарів мастильного матеріалу, що призводить до безпосередньої механодеструкції граничного шару в зоні проковзування контактних поверхонь, з урахуванням пружно-фрикційної взаємодії граничного шару мастильного матеріалу з приповерхневим шаром металу.

**Ключові слова:** wear, самоорганізація, змащування, деформація, граничні шари, дотичні зусилля, шорстка поверхня.